Section 1 SIR model

It is well known that SIR is good model for explaining disease spread like COVID-19. Politicians would like to reduce ratio of infection in some period in order not to destroy medical system. For this purpose, they would like to control one of the essential parameter β (infection ratio: it describes number of person which one positive person infects one day).

SIR-model ([1]) was developed around one hundred years ago to analyze Spanish Flu spread. In this SIR model, all population is divided into three kinds:

$S(t)$: Susceptible,
$I(t)$: Infected
$R(t)$: Recovered or Removed

We assume total population $N = S(0)$. The susceptible individuals (S) change into infected individuals (I) proportionate to $\frac{S}{N} \times I$ with ratio β. Thus, we can say:

β : the infection ratio, It represents the ratio of transmitting disease from one infectious individual ([2]). It can be translated number of new infected individuals which is transmitted from one infected person per day.

γ : the recovery factor (or rate). It is the ratio an infected recovers and moves into the resistant phase. The infected individuals change into recovered individuals (R) and infected period follows exponential distribution. Roughly saying, it means the probability which infected individuals recovering in one certain period. It is also said the quarantine (or isolation) ratio. At the very beginning of epidemic, quarantine action work mainly on reducing inclement of $I(t)$.

If we use continuous number to describe person number, the model equations are the following:

$$\begin{cases} \frac{dS}{dt}(t) = -\frac{\beta}{N} S(t)I(t) \\ \frac{dI}{dt}(t) = \frac{\beta}{N} S(t)I(t) - \gamma\, I(t). \\ \frac{dR}{dt}(t) = \gamma\, I(t) \end{cases}$$

If one normalizes each unknown (i.e., $s(t) = \frac{S(t)}{N}$, $i(t) = \frac{I(t)}{N}$, $r(t) = \frac{R(t)}{N}$), we have

$$\begin{cases} \frac{ds}{dt}(t) = -\beta\, s(t)i(t) \\ \frac{di}{dt}(t) = \beta\, s(t)i(t) - \gamma\, i(t). \\ \frac{dr}{dt}(t) = \gamma\, i(t) \end{cases} \quad (1.1)$$

Please note that the equality:

$$\frac{d}{dt}(s(t) + i(t) + r(t)) = 0$$

holds. This means total population is constant (no one died or going out and no one birth or coming in).

From the second equation of (1.1), if we assume β is equal to zero, it is easy to see each individual infection period is assumed to follow exponential distribution and $1/\gamma$ becomes mean infection period.

Problem (Solving Equation)

We can solve the equation (1.1) numerically with the appropriate choice of β and γ and initial data $s(0) = s_0$, $i(0) = i_0$, $R(0) = r_0$.

The model has several important parameters (or quantity) which the behavior of the solution changes dramatically. For example, if one look at the second equation of (1.1), it is easy to see the sign of $\beta\, s(t)i(t) - \gamma\, i(t)$ plays an essential roll. If it is positive $i(t)$ increases, if negative it decreases.

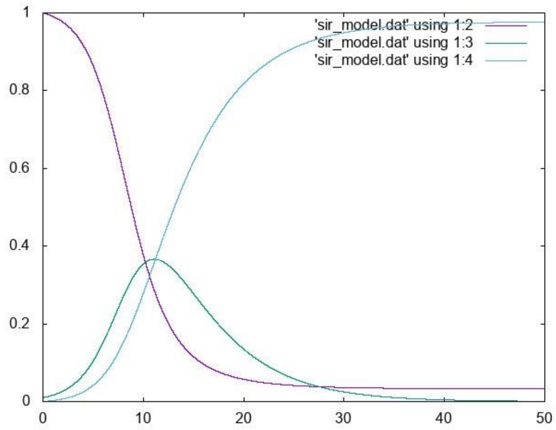
If, at some time $t = t_0$, $(\beta s(t_0) - \gamma)\, i(t_0)$, is positive, infected individuals are increasing. So, we call $\mathcal{R}_0 = \beta s(t_0)/\gamma$ effective reproduction number (基本再生産数).

基本再生産数は 1 が境目で感染者が増大するか減少するかどうか決まる。

So, the researchers are always taking care of this number and want to control it. They need to determine it by observing real data. Here we need to take care that the parameter γ is controllable by quarantine (or removing) some percentage of person artificially from the system. This is the reason why they request government to lockdown cities. Of course, the locking down action effects on $s(t_0)$, $i(t_0)$ and $r(t_0)$. (Note that they are not independent.)

- Example

$\beta = 0.5, \gamma = 0.2, S_0 = 1.0, I_0 = 0.01, R_0 = 0.0, \Delta t = 0.001, n = 50000.$



References
[1] W. O. Kermack and A. G. McKendrick (1927). "A Contribution to the Mathematical Theory of Epidemics". Proc. Roy. Soc. of London. Series A 115 (772): 700-721.
[2] https://idmod.org/docs/general/model-sir.html

Section 2　Fixing parameters from real medical data

In this section, we will consider how to determine β (infection ratio). It is determined not only from biological observation. For example, social action such as lockdown policy (restriction of person to go out) sometimes works essential to this number. So, we should carefully extract this number from statistical social data. The difficulty is coming from:

(1) existence of undetected infected individuals,

(2) lack of reliability of testing method to detect infected individuals.

As we know, only by checking the percentage of people who got immunity（免疫）, we cannot know the number of infected individuals because of instability of immunity.

Here, we shall focus on the increment of daily new infected individuals (judged by so called PCR test). We can get this information from Japanese government home page. They are gathering the results of PCR test from over Japan every day. Since human beings can't detect all positives, but if the testing method is always same, it could be a good sampling method. So we can know trend and obtain the feature of graph $i(t)$ especially the peak points and inflection points（変曲点）.

It is important to find a peak of $I(t)$'s graph to calculate parameters.

Problem 2.1

Gather data of every day's increment of positive individual number from Japanese government's home page (for example:
https://www.mhlw.go.jp/stf/seisakunitsuite/bunya/0000121431_00086.html)
and draw graph of $I(t)$ (or $i(t)$). Moreover you should fix parameters such as β or $\mathcal{R}_0$.

Hints:

(1) you may assume $\gamma = 1/7$.

(2) by using the inflection point obtained from real data, you may fix β by graph fitting.

(the graph can be obtained from the solution of SIR with β by the appropriate choice of norm measuring distance between the graph and real data.)

Problem 2.2

Using parameter obtained above, draw a graph of $s(t)$, $i(t)$, $r(t)$.

Section 3　Death control problem

All government of the world are facing difficult problem 'when shall we stop lock down policy'. Each policy eventually converges to control problem of β. But we can't control it explicitly. Just we may know, after several weeks, the relation between policy and β.

But in this section, we assume that we can control β for the sake of simplicity. Here we introduce new parameter

α :　the coefficient of contact reducing(接触削減係数). It is used by multiplying to β. So $\alpha\beta$ is new infection ratio and α is assumed to be controllable. If α is one, politically nothing to be done and it is zero, complete lock down policy is implemented.

It is well known by Okun's law (Arthur Okun 1962) that the unemployment rate (which is compared with previous period)(失業率前期比) and GDP growth rate (実質 GDP 成長率) has strong minus correlation. Please note that correlation constants are depending on country.

Student can consult web page blow;

https://toyokeizai.net/articles/-/344140?page=2

It is written in Japanese but I think you can understand the meaning of graph. Its horizontal axis is GDP growth rate (実質 GDP 成長率) and vertical axis is difference between previous and the present year measured by %.

In Japan, it is also well known that unemployment rate and suicide rate to population has strong correlation. Students can see the graph:

https://www.newsweekjapan.jp/stories/world/2019/01/post-11515_1.php

by Newsweek Japan.

So we must consider severe problem like below.

Problem 3.1

Minimize

(corona death) + (economic death)

by controlling parameter α and multiplying timing. You can assume that α controls day by day's GDP.

At the beginning, we can assume that α also controls day by day GDP linearly. You can assume the Japanese data

GDP　500,000,000,000,000 JPN(500 兆円) per year

Population　120,000,000 (1 億 2 千万人)

How to use alpha for calculating GDP reduction?

(daily GDP) times (alpha) times (day) = reduction of GDP

SIR model is not including death. But the number of dead is so small in Japan, I think no need to change the model.

Students can consult the ratio of death against infected individuals which you can see government official cite.

We will consider how to apply α_policy. As we defined already, α is reduction ratio of economic work (for example, reduction of frequency of people's going out. We will apply policy at some time ($t = 0$) and end it at ($t = N$). Totally, $N + 1$ days application. After ending policy, we return $\alpha = 1$. The system will be expressed in the following:

$$
(\mathrm{P}_{\alpha,N}) \begin{cases} \frac{dS}{dt}(t) = -\tilde{\alpha}_N(t)\beta\, S(t)I(t) \\ \frac{dI}{dt}(t) = \tilde{\alpha}_N(t)\beta\, S(t)I(t) - \gamma\, I(t), \quad t \in (0,\, T] \\ \frac{dR}{dt}(t) = \gamma\, I(t) \end{cases}
$$

$$
\tilde{\alpha}_N(t) = \begin{cases} \alpha, & t \in [0,\, N] \\ 1, & t \in (N,\, T]. \end{cases}
$$

α : the coefficient of contact reducing $\in [0,\, 1]$ (policy).

$\alpha = 1$ → politically nothing to be done

$\alpha = 0$ → complete lock down

N : the term under the policy α.

(Total people number) $= S(0) + I(0) + R(0) \equiv S(t) + I(t) + R(t)$.

(corona death) $S_{\alpha,N}$, $I_{\alpha,N}$, $R_{\alpha,N}$: the solutions for ($\mathrm{P}_{\alpha,N}$)

$$R_{\alpha,N}(T) \times \eta$$

where η : the case fatality rate(CFR) You may assume $\eta = 0.01$.

(economic death) number of suicide people in <u>2021</u> would be estimated by

$$(\text{Total people number}) \times Y(\alpha)$$

where $Y(\alpha)$: Suicide rate in Japan under α-policy given by;

$$Y(\alpha) = 1.949\,(-0.1078\,Z(\alpha) - 0.0803\,) + 14.345$$

where $\mathrm{Z}(\alpha)$: GDP growth rate given by;

$$Z(\alpha) = \frac{(\alpha\text{-policy GDP:this year}) - (\text{GDP:last year})}{(\text{GDP:last year})}$$