Derivation of heat equation

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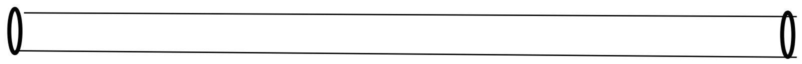
- Setting： Thin metal rod(1 dim.), ignore radiation(放射)
- Focus the calorie(熱量) to observe heat conduction
- Measurable thing is temperature(温度)
- Heat flow(熱の流量)is proportional to temperature gradient (Fourier’s law)
- $u(x,t)$: temperature distribution (温度分布)



Thin metal rod

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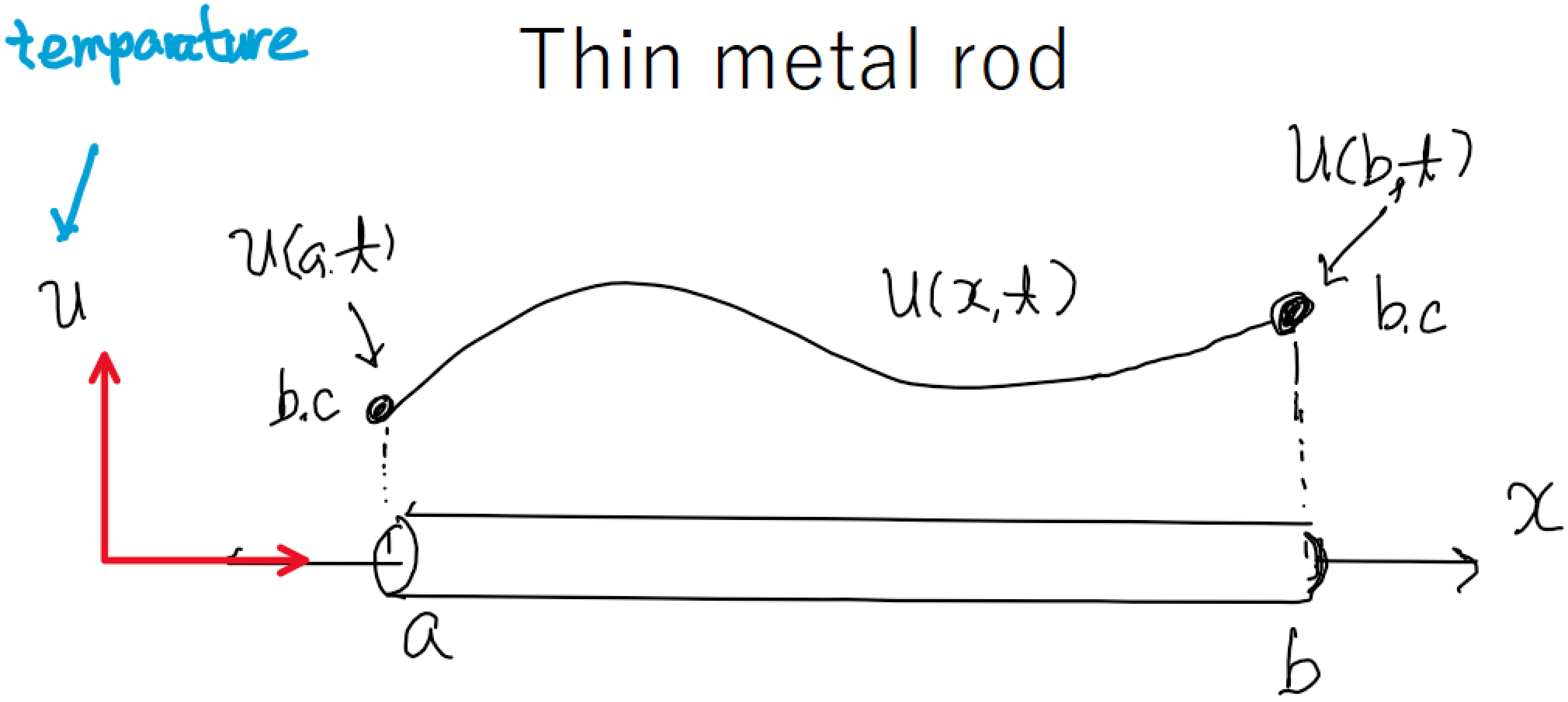

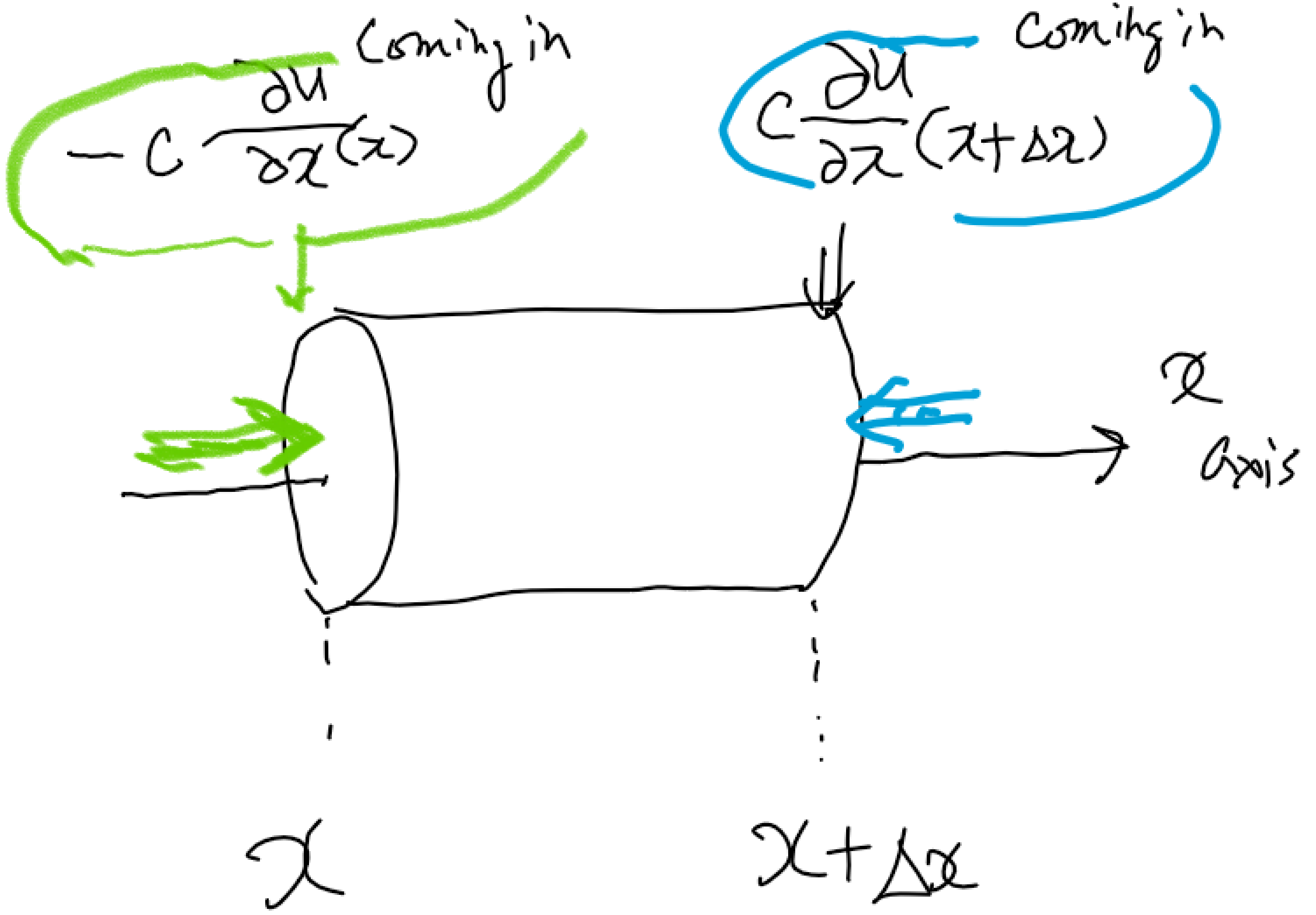
coming in
$-c\frac{\partial u}{\partial x}(x)$
coming in
$c\frac{\partial u}{\partial x}(x+\Delta x)$
x
axis
x
x+Δx

- Consider heat flow coming into the small domain $(x, x + dx)$
- Heat flow(left at x) $= -c\frac{\partial u}{\partial x}(x,t)$.
- Heat flow(left at $x + dx$) $= c\frac{\partial u}{\partial x}(x + dx, t)$.

Calorie increment is proportionate to heat flux coming into the domain. So, we have

$$dx\frac{\partial u}{\partial t}(x,t) = c\left(\frac{\partial u}{\partial x}(x + dx, t) - \frac{\partial u}{\partial x}(x,t)\right).$$

Dividing dx, and $dx \to \mathbf{0}$, we got

$$\frac{\partial u}{\partial t}(x,t) = c\frac{\partial^2 u}{\partial x^2}(x,t).$$

Heat flux

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- Heat flow(left at x) $= -c\frac{\partial u}{\partial x}(x,t)$.
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